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Aharonov–Bohm-type electron interference in the presence of one-mode SU(1,1) coherent state

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Abstract. Dynamic behaviour of Aharonov–Bohm-type electron interference in the presence of a nonclassical electromagnetic field is investigated. The visibility of the time-averaged interference pattern is discussed for SU(1,1) coherent state (CS) and a comparison with other states is made. It is shown that the dynamic behaviour of the electron interference exhibits collapse and revival (CR) phenomenon for SU(1,1) CS. It is also shown that CR phenomenon of electron interference is closely related to the fluctuation of a nonclassical electromagnetic field.

1. Introduction

The Aharonov–Bohm (AB) [1, 2] effect, both electric and magnetic, has been studied extensively for a long time. In this article our discussion is only related to the magnetic AB effect, i.e. the production of a relative phase shift between two electron beams enclosing a magnetostatic flux even if the electron beams are prevented from entering the region of the magnetic flux. The effects of enclosed fluxes often appear as observable changes in quantum interference patterns, although the fluxes may also affect the energy spectrum and kinetic momentum eigenvalues of the electrons. The AB effect is usually explained by means of the vector potential, which is present in multiply connected regions of space where no magnetic induction field acts on the electrons, whereas the charge and current densities are unique, the vector potentials are susceptible to gauge transformation. Nevertheless, the observable AB phase shifts are gauge invariant, depending only on the magnetic flux in the region from which the electron is excluded. Such an effect is inconceivable in classical physics and directly demonstrates the gauge principle of electromagnetism [3]. Most of the relevant properties of the quantum effects of fluxes can be discussed in terms of the two-slit interference experiment with electrons. Evidence for the AB effect have been found by Lischke [4] and Tonomura *et al* [5] in such experiments.

Recently electron interference in the presence of nonclassical electromagnetic fields have been studied [6], and the visibility of the time-averaged intensity has been discussed in a similar way to the Shapiro steps in the context of Josephson junction [7]. A comparison with the corresponding classical case has also been made. In the last few years there have been a lot of theoretical and experimental works on nonclassical electromagnetic fields, and those studies reveal many properties which are due to the quantum nature of a nonclassical electromagnetic field and cannot be understood classically, such as squeezing, sub-Possionian photon statistics and oscillation of the photon-number distributions etc [8]. It is very interesting to study the effect of a nonclassical magnetic flux on electron interference.

In this case the relative phase shift between the two electron beams is a quantum-mechanical operator, whose expectation value with regard to the density matrix describing the nonclassical electromagnetic field is time-dependent and has quantum fluctuations in origin. The main idea we have in mind is that the quantum fluctuations of nonclassical electromagnetic fields is expected to play a destructive role on the interference of the electrons and we consider some effects, which have no counterpart in the case of a classical electromagnetic field, that might be exhibited in such systems. In this article we will reveal such effects, that is, the collapse and revival (CR) phenomenon in electron interference. The so-called CR phenomenon is well known in the context of Jaynes–Cumming's (JC) model in quantum optics [9–11]. A two-level atom interacting with a nonclassical electromagnetic field is known as a JC model [9], in which Eberly *et al* [10] have theoretically found CR in the time evolution of atomic inversion. Evidence for CR has also been found by Rempe *et al* [11].

2. Electron interference in the presence of a nonclassical electromagnetic field

The derivation of the expressions for the phase shift in magnetic AB effects is well known but for the convenience of our discussion we briefly summarize the main results. In the usual AB experiment, an electron wave is split into two coherent waves. They pass on opposite sides of a solenoid and then recombine. The wavefunction of the electron splits into two parts $\psi = \psi_1 + \psi_2$, where ψ_1 represents the beam on one side of the solenoid and ψ_2 the beam on the opposite side. Each of these beams stays in an opposite simply connected region. (In the following we will use the system of units in which $\hbar = c = k_B = 1$, and the charge of the electron is dimensionless and is equal to $e = \sqrt{4\pi/137}$.) We can write $\psi_1 = \psi_1^0 \mathrm{e}^{-\mathrm{i}S_1}$, $\psi_2 = \psi_2^0 \mathrm{e}^{-\mathrm{i}S_2}$, where S_1 and S_2 are equal to $e\int \boldsymbol{A}\cdot \mathrm{d}\boldsymbol{l}$ along the paths of the first and second beams, respectively, and ψ_1^0 and ψ_2^0 are the wavefunctions of the electrons when $A = 0$. The intensity of the electrons at some point R on the screen is

$$I(R) = |\psi_1 + \psi_2| = |\psi_1|^2 + |\psi_2|^2 + 2|\psi_1\psi_2| \mathrm{Re}\,\{\exp[\mathrm{i}(\sigma + \Delta S)]\} \tag{1}$$

where $\sigma = \arg(\psi_1^0) - \arg(\psi_2^0)$ and is a function of R. As the point R moves along the screen, the phase difference σ changes, and we obtain the interference pattern. The interference between the two beams will evidently also depend on the phase shift ΔS. The phase shift ΔS, in this case is given by

$$\Delta S = e \oint \boldsymbol{A}\cdot \mathrm{d}\boldsymbol{l} = e\phi_0. \tag{2}$$

Here, the integral is carried out along a closed curve connecting the two paths. Although the magnetic field B is zero everywhere outside the solenoid, the vector potential $\boldsymbol{A}$ cannot vanish there. This is because the loop integral of $\boldsymbol{A}$ around the solenoid is equal to the magnetic flux $\phi_0 = \int \boldsymbol{B}\cdot \mathrm{d}\boldsymbol{s}$ inside it. Therefore a nonzero phase shift can be observed.

An experiment in the case of nonclassical electromagnetic fields at a low temperature similar to the usual AB experiment was proposed in [6]. The low-temperature requirement makes the thermal fluctuations smaller than the quantum fluctuations. In the experiment a beam of electrons is split into two (for example, by using an electrostatic biprism) and then each of the beams enters a waveguide through one hole and exits through another, respectively, in which the nonclassical electromagnetic field is travelling (see [6, figure 2]). The magnetic field is perpendicular to the plane of the paths of the two beams, while the electric field is on the plane. In such an experiment the electrons feel both an AC vector potential A and an AC electric field E. Integration of A and E in a closed loop around the flux gives the magnetic flux ϕ, and the electromotive force V, correspondingly. Now ϕ and V are quantum operators and obey the commutation relation:

$$[\phi, V] = \mathrm{i}\omega \tag{3}$$

and

$$a = 2^{-1/2}[\phi + \mathrm{i}\omega^{-1}V] \tag{4}$$

$$a^{+} = 2^{-1/2}[\phi - \mathrm{i}\omega^{-1}V] \tag{5}$$

$$[a, a^{+}] = 1 \tag{6}$$

where a^{+}, a are the corresponding creation and annihilation operators of the nonclassical electromagnetic field with frequency ω. The Hamiltonian of a one-mode electromagnetic field is assumed to be

$$H = \omega(a^{+}a + \tfrac{1}{2}). \tag{7}$$

Using the Hamiltonian (7), we can obtain ϕ in the Heisenberg picture as follows

$$\phi(t) = 2^{-1/2}[\exp(\mathrm{i}\omega t)a^{+} + \exp(-\mathrm{i}\omega t)a]. \tag{8}$$

Here the external field is treated as free for the case of weak electron currents. In the case of a nonclassical electromagnetic field, equation (1) becomes

$$I(R, t) = \mathrm{Tr}\{\rho|\psi_1 + \psi_2|^2\} = |\psi_1|^2 + |\psi_2|^2 + 2|\psi_1\psi_2|\mathrm{Re}\,\{\exp(\mathrm{i}\sigma)\,\mathrm{Tr}[\rho\exp[\mathrm{i}e\phi(t)]]\} \tag{9}$$

where ρ is the density matrix of the nonclassical electromagnetic field. Apart from the nonclassical electromagnetic field, a classical flux $V_0 t$ is also imposed which produces a static electromotive force V_0. This can be achieved by using a solenoid with a current that increases linearly as a function of time. Then from equations (8) and (9) we obtain

$$\begin{aligned} I(R, t) = |\psi_1|^2 + |\psi_2|^2 + 2|\psi_1\psi_2|\mathrm{Re}\,\{&\exp[\mathrm{i}(\sigma + eV_0 t)] \\ &\times \mathrm{Tr}[\rho D[2^{-1/2}e\exp(\mathrm{i}\omega t + \mathrm{i}\pi/2)]]\} \end{aligned} \tag{10}$$

where $D(A) = \exp(Aa^{+} - A^{*}a)$ is the displacement operator.

In [6] several examples of density matrixes ρ are considered and it is concluded that the time-averaged interference fringes exist only for special values of V_0 which depend on the nature of the applied nonclassical electromagnetic field. An SU(1,1) coherent state (CS) field is another type of nonclassical field which has been widely studied [12], and in this paper we are interested in the effects of its nonclassical characteristics on the dynamic behaviour of electron interference. The realization for the SU(1,1) Lie algebra for a single mode is $K_0 = (a^{+}a + aa^{+})/4$, $K_{+} = a^{+2}/2$, and $K_{-} = a^{2}/2$. An SU(1,1) CS is defined as $S(\beta)|q, k\rangle$, where

$$S(\beta) = \exp(\tfrac{1}{2}\beta^{*}a^{2} - \tfrac{1}{2}\beta a^{+2}) \tag{11}$$

is the squeeze operator with $\beta = |\beta|\mathrm{e}^{-\mathrm{i}\varphi}$, and $|q, k\rangle$ is the basis with $k = \frac{1}{4}$ or $k = \frac{3}{4}$, $q = 0, 1, 2, \ldots$. For this state the Hilbert space $|n\rangle$, where $|n\rangle$ is the usual number state, is split into two spaces, one for $k = \frac{1}{4}$, $n = 2q$ (even), and one for $k = \frac{3}{4}$, $n = 2q+1$ (odd). In fact all the squeezed number states (SN) can be interpreted as generalized SU(1,1) CS, which are generated by applying the operator $S(\beta)$ to any state for $k = \frac{1}{4}$ or $\frac{3}{4}$. For simplicity the generalized SU(1,1) CS, or the SN, $S(\beta)|m\rangle$, is considered in the following where $|m\rangle$ denotes the usual number state and may be produced by the action of a nondegenerate parametric amplifier on the number state [12]. The photon number distribution for this state is oscillatory with zero probability for odd n (m = even) or for even n (m = odd). The squeeze operator $S(\beta)$ transforms the annihilation operator as follows

$$S^{+}(\beta)aS(\beta) = \cosh(|\beta|)a - \exp(-\mathrm{i}\varphi)\sinh(|\beta|)a^{+}. \tag{12}$$

For a pure SN the density matrix $\rho = S(\beta)|m\rangle\langle m|S^{+}(\beta)$, and it can be derived from equations (10) and (12) that

$$\begin{aligned} I(R,t) &= 1 + \mathrm{Re}\,\{\exp[\mathrm{i}(\sigma + eV_0 t)] \\ &\quad \times \sum_n \langle n|S(\beta)|m\rangle\langle m|S^{+}(\beta)D[2^{-1/2}e\exp(\mathrm{i}\omega t + \mathrm{i}\pi/2)]|n\rangle\} \\ &= 1 + \mathrm{Re}\,\{\exp[\mathrm{i}(\sigma + eV_0 t)]\langle m|S^{+}(\beta)D[2^{-1/2}e\exp(\mathrm{i}\omega t + \mathrm{i}\pi/2)]S(\beta)|m\rangle\} \\ &= 1 + \mathrm{Re}\,\{\exp[\mathrm{i}(\sigma + eV_0 t)]\exp[-Y(t)]L_m[2Y(t)]\} \end{aligned} \tag{13}$$

where $L_m(x)$ are the Laguerre polynomials and

$$Y(t) = \frac{e^2}{4}[\cosh(2|\beta|) - \sinh(2|\beta|)\cos(2\omega t + \varphi)]. \tag{14}$$

In deriving equation (13) we have used the formula

$$\langle m|D(A)|m\rangle = \exp(-|A|^2/2)L_m(|A|^2) \tag{15}$$

and for simplicity we have assumed that $|\psi_1| = |\psi_2| = 1/\sqrt{2}$. Using formula $\exp(A\cos\theta) = \sum_{n=-\infty}^{\infty} I_n(A)\exp(\mathrm{i}n\theta)$ we can expand equation (13) into

$$\begin{aligned} I(R,t) &= 1 + \exp[-e^2\cosh(2|\beta|)/4]\mathrm{Re} \\ &\quad \times \left\{ \sum_{n=-\infty}^{\infty} I_n[e^2\sinh(2|\beta|)/4]\exp\{\mathrm{i}[(eV_0 + 2n\omega)t + \sigma + n\varphi]\}L_m[2Y(t)] \right\} \end{aligned} \tag{16}$$

where $I_n(x)$ is the modified Bessel function. Noting that $L_m(x) = \sum_{k=0}^{m} \binom{m}{k} (-x)^k/k!$ [13], taking into account the following relations: $(a+b)^k = \sum_{l=0}^{k} \binom{k}{l} a^l b^{k-l}$, $\cos x = (\mathrm{e}^{\mathrm{i}x} + \mathrm{e}^{-\mathrm{i}x})/2$, and after some trivial transformations, we can rewrite equation (13) as

$$\begin{aligned} I(R,t) &= 1 + \mathrm{Re}\left\{ \exp[-e^2\cosh(2|\beta|)/4] \sum_{n=-\infty}^{\infty}\sum_{k=0}^{m}\sum_{l=0}^{k}\sum_{p=0}^{l} I_n[e^2\sinh(2|\beta|)/4] \right. \\ &\quad \left. \times \binom{m}{k}\binom{k}{l}\binom{l}{p}\left(\frac{1}{2}\right)^l L^{k-l}M^l/k!\,\mathrm{e}^{\mathrm{i}\{[2(2p-l+n)\omega + eV_0]t + \sigma + (2p-l+n)\varphi\}} \right\} \end{aligned} \tag{17}$$

where $M = e^2\sinh(2|\beta|)/2$, $L = -e^2\cosh(2|\beta|)/2$. It is easy to see from equation (17) that when the following condition is satified

$$eV_0 = 2N\omega \tag{18}$$

we can take the time average of equation (17) and obtain

$$\begin{aligned} \overline{I(R)} &= 1 + \exp[-e^2\cosh(2|\beta|)/4]\sum_{k=0}^{m}\sum_{l=0}^{k}\sum_{s=0}^{l} I_{l-N-2s}[e^2\sinh(2|\beta|)/4] \\ &\quad \times \binom{m}{k}\binom{k}{l}\binom{l}{s}\left(\frac{1}{2}\right)^l L^{k-l}M^l/k!\cos(N\varphi - \sigma). \end{aligned} \tag{19}$$

The phase difference σ changes with R, so it is easy to see from equation (19) that as R moves along the screen we obtain an interference pattern. The visibility of the interference fringes can be derived from equation (19)

$$\alpha = \exp[-e^2\cosh(2|\beta|)/4]\sum_{k=0}^{m}\sum_{l=0}^{k}\sum_{s=0}^{l} I_{l-N-2s}[e^2\sinh(2|\beta|)/4] \times \binom{m}{k}\binom{k}{l}\binom{l}{s}\left(\frac{1}{2}\right)^l L^{k-l}M^l/k!. \tag{20}$$

When equation (18) is not satisfied, time averaging will destroy the interference pattern.

From [6] we learn that the voltage steps for squeezed-vacuum states are double in size in comparison with the voltage steps for coherent and squeezed states. From equation (18) we can conclude that the voltage steps for SN are the same as the voltage steps for squeezed-vacuum states which are double in size in comparison with the voltage steps for coherent and squeezed states. Actually squeezed-vacuum states are just a special case of SN for $m = 0$.

3. Collapse and revival in the electron interference

With regards to the usual AB experiment, electron interference in the presence of a classical magnetostatic flux has been fully discussed, and in this case the flux is definitely without fluctuations. In the case of a nonclassical electromagnetic field the flux is a quantum operator whose expectation value has quantum fluctuations. It is the quantum fluctuations of flux ϕ that will partially destroy the electron interference. To see how the quantum noise of nonclassical electromagnetic fields affect the phase shift and hence the corresponding electron interference, we will study the time evolution of electron intensity.

We plot the time evolution of electron intensity versus the scaled time $\omega t/\pi$ for various values of $|\beta|$ and m, and for fixed R and other parameters in figure 1. In numerical calculations we found that when $|\beta|$ is small, the oscillations of electron intensity appear to be almost regular, with no true CR; when $|\beta|$ is increased, we notice that CR occurs with incomplete collapses and essentially complete revivals as figure 1(a) shows; when $|\beta|$ is large enough, CR occurs with complete collapse, as figures 1(b) and (c) show; with increasing $|\beta|$ CR becomes more and more compact and distinct, and the time between revivals also increases. For $m = 0$, i.e. the squeezed-vacuum state, the frequency is eV_0 during the revivals, which means that perfect and complete oscillations are indeed essentially sinusoidal as figures 1(a) and (b) show. As m is increased, we observe increasingly irregular behaviour of the oscillations, see figure 1(c).

It can be inferred from equation (19) that for $\varphi = 0$ when $\sigma(R_1) = 0$, $\overline{I(R_1)}$ reaches its maximum $1 + \alpha$; when $\sigma(R_0) = \pi/2$, $\overline{I(R_0)} = 1$; and when $\sigma(R_2) = \pi$, $\overline{I(R_2)}$ reaches its minimum $1 - \alpha$, where α is the visibility. In figure 2 we plot the time evolution of $I(R, t)$ for $\sigma(R_1) = 0$ and $\sigma(R_2) = \pi$, using the same parameters as in figure 1(c). Comparing figure 1(c) with figure 2 we can see that $I(R, t)$ simultaneously collapse to 1 in the collapse period independent of $\sigma(R)$, this means that although the time-averaged interference pattern is unchanged, it will completely disappear in this period. Then in the revival period, the time-averaged electron intensities over this period are bigger than 1 for $\sigma(R_1) = 0$ (see figure 2(a)), less than 1 for $\sigma(R_2) = \pi$ (see figure 2(b)) and equal to 1 for $\sigma(R_0) = \pi/2$ (see figure 1(c)). This means that the interference pattern reappears and the process is periodically repeated. So, if we are able to observe the time-dependent interference pattern (this might be realized with a clever stroboscopic measurement in the spirit of [14]) we would be able to directly observe the CR in the experiment.

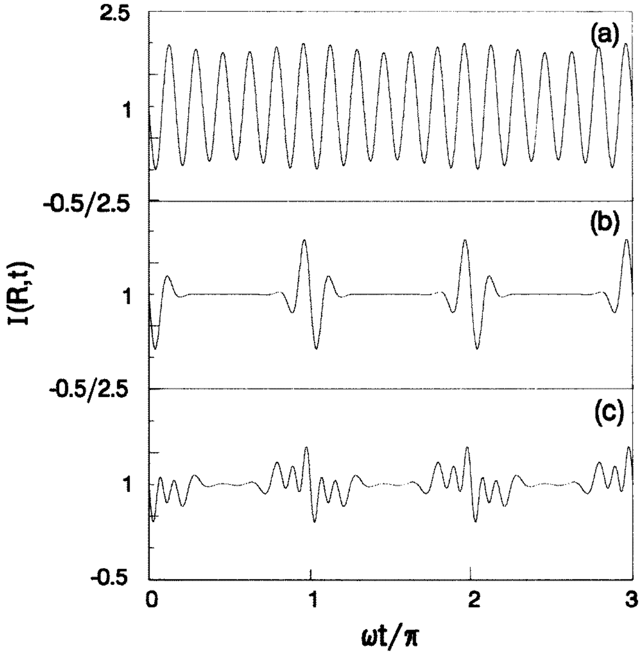

Figure 1. Plot of electron intensity versus scaled time $\omega t/\pi$ for $\sigma(R_0) = \pi/2$, $\varphi = 0$, $eV_0/\omega = 12$, and for, (a) $|\beta| = 1$, $m = 0$; (b) $|\beta| = 3$, $m = 0$; (c) $|\beta| = 3$, $m = 2$.

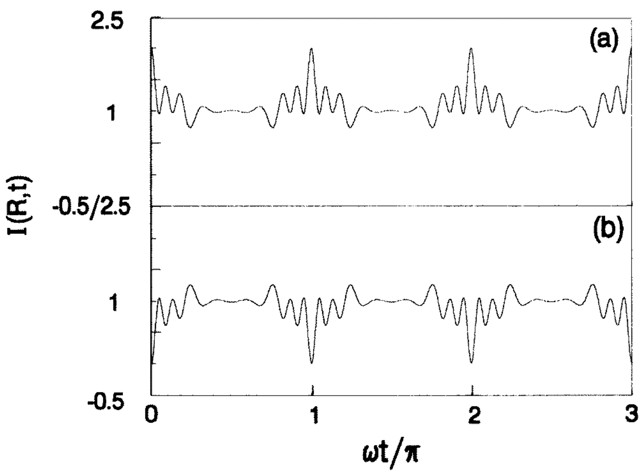

Figure 2. Plot of electron intensity versus scaled time $\omega t/\pi$ for $|\beta| = 3$, $\varphi = 0$, $eV_0/\omega = 12$, $m = 2$, and for (a) $\sigma = 0$; (b) $\sigma = \pi$.

The collapses are easily understood. It can be seen from equation (14) that $Y(t)$ is always bigger than zero and is an oscillating function with a period of π/ω. Thus the second term in equation (13), which represents the interference between the two electron beams and determines the interference pattern, is weighted with a time-dependent factor which periodically supresses the coherence of the electrons. The factor $\exp[-Y(t)]$ is intimately related to the quantum fluctuation of the nonclassical electromagnetic field and causes partial destruction of the interference. The maximum of $Y(t)$ is $Y_{\max} = e^2[\cosh(2|\beta|) + \sinh(2|\beta|)]/4$ for fixed $|\beta|$. For small $|\beta|$ the minimum of $\exp[-Y(t)](= \exp(-Y_{\max}))$ is not small enough, we obtain the incomplete collapse as figure 1(*a*) shows. When $|\beta|$ is large enough, CR occurs with complete collapse, and now $\cosh(2|\beta|) \simeq \sinh(2|\beta|) \simeq \exp(2|\beta|)/2$, the collapse function can be found from equations (13) and (14) as

$$\exp\left[-\frac{e^2}{4}\exp(2|\beta|)\sin^2(\omega t + \varphi/2)\right] \tag{21}$$

and the collapse time τ_c can also be found from equation (21)

$$\tau_c = 2\exp(-|\beta|)/(\omega e). \tag{22}$$

It is clear that when $|\beta|$ is increased, the collapse time becomes shorter, which is proportional to $\exp(-|\beta|)$ while the period of CR is fixed to unity in the scaled time. For large $|\beta|$, equations (21) and (22) are consistent with the numerical calculations.

CR phenomenon of electron interference can be explained as a consequence of quantum interference in phase space. One can see that the summation in equation (17) represents oscillations with different freqencies due to the effect of a nonclassical electromagnetic field. Because different components in the summation oscillate with different frequencies, they will become decorrelated with a range of the frequency that makes a significant contribution to the sum, i.e. the collapses are due to the destructive interference of oscillations with different frequencies in equation (17). The revivals are a manifestation of the quantum nature of a nonclassical electromagnetic field, which is mathematically reflected in the discrete summation, that means the evolution of the electron intensity is determined by the individual field quanta. The discrete characteristic ensures that after some finite time all oscillating terms almost come back in phase with each other, restore the coherent oscillations, and give an appreciable value to electron intensity.

When $V_0 = 0$, the electron intensity can also exhibit CR phenomenon for this state, as figure 3 shows. When a nonclassical electromagnetic field is not present, the voltage V_0 will bring about an oscillation with frequency eV_0. When both V_0 and a nonclassical electromagnetic field are present, the interaction between the oscillation with frequency eV_0

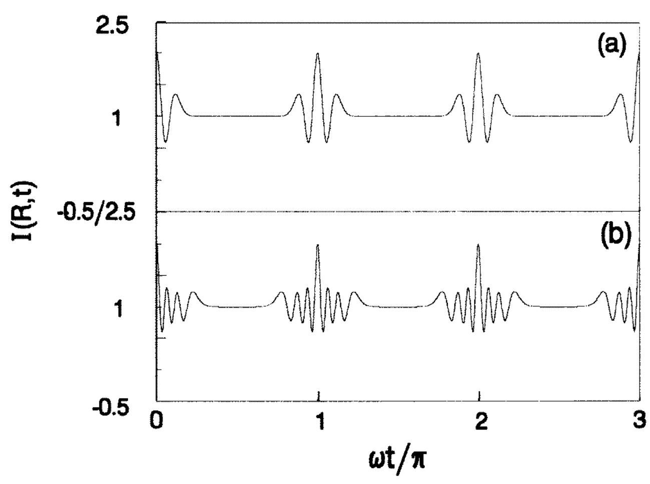

Figure 3. Plot of electron intensity versus scaled time $\omega t/\pi$ for $\sigma = 0$, $\varphi = 0$, $V_0 = 0$, $|\beta| = 3.5$, and for, (*a*) $m = 2$, (*b*) $m = 6$.

and the oscillation due to the nonclassical electromagnetic field leads to the appearance of CR phenomenon and the time-averaged interference pattern with a proper value of V_0. However, when $V_0 = 0$, the external nonclassical electromagnetic field brings about an oscillation with a basic frequency of the external field and its harmonics, which can be easily seen from equation (17) with $V_0 = 0$. In this case it is the contribution of the coherence of the infinite harmonics that bring about CR phenomenon, as figure 3 shows, and CR is entirely due to the coherence of the external field.

It is worth noting that the number state is a special case of SN for $|\beta| = 0$. For the number state from equation (13) we can easily obtain

$$I(R,t) = 1 + \exp(-e^2/4)L_m(e^2/2)\cos(eV_0t + \sigma). \tag{23}$$

It is apparent that there is no CR phenomenon that exists for this state. The number state and the classical field share the property of a definite intensity which is needed to avoid the interferences leading to a collapse. For number states the photon number is definite, but the phase is indefinite, and is closely related to the particle nature of radiation rather than its wave nature. By squeezing the number state the determinstic photon number is blurred, and the quadrature is squeezed in one direction, and the state becomes phase dependent.

4. Quantum fluctuations in electron interference

As mentioned above, in the case of a nonclassical electromagnetic field ϕ is a quantum operator whose expectation value has quantum fluctuations. Now we are interested in the quantum fluctuations of the operator $f \equiv \mathrm{Re}\,\{\exp[\mathrm{i}(e\phi+eV_0t+\sigma)]\}$ whose expectation value determines the dynamics of the electron interference, and we will show in the following that the CR phenomenon of an electron interference is closely related to the fluctuation of such an operator. The fluctuation of the operator f is defined as

$$\langle(\Delta f)^2\rangle = \langle f^2\rangle - \langle f\rangle^2. \tag{24}$$

For SN from equations (8), (12), (15) and (24) we can obtain

$$\langle(\Delta f)^2\rangle = \tfrac{1}{2} + \tfrac{1}{2}\exp[-4Y(t)]L_m[8Y(t)]\cos(2eV_0t + 2\sigma) \\ - \exp[-2Y(t)]L_m^2[2Y(t)]\cos^2(eV_0t + \sigma). \tag{25}$$

In figure 4 we display some examples of time evolutions of quantum fluctuations of operator f for the same parameters as figure 1 for SN. From numerical calculation we reach the following conclusion: the time evolution of the electron interference brings about fluctuation reduction in f, the minimum fluctuations of f occurring when the electron intensity is revived to its maximum; the maximum of fluctuations is achieved at the collapse period, where electron intensity reaches its steady value, which is equal to 1 at any R on the screen (now $I(R,t)$ is independent of σ or R) that is, the complete destruction of the interference fringe. This can be easily understood, because the fluctuations have their origin in the destructiveness of quantum interference of different oscillations, i.e. a rise in fluctuations means that different oscillations begin to partially lose their correlations, and when the fluctuations are smallest, different oscillations are mostly correlated.

5. Conclusion

We have investigated the dynamic behaviour of AB-type electron interference in the presence of a nonclassical electromagnetic field. In this case the relative phase shift between the two electron beams is a quantum operator and its expectation value determines the dynamics of

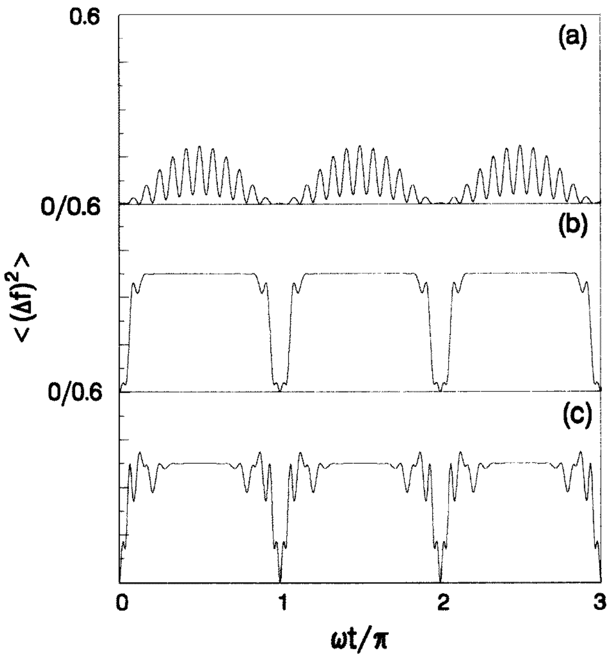

Figure 4. Plot of the fluctuation of the operator f, $\langle(\Delta f)^2\rangle$, versus scaled time $\omega t/\pi$ for the same parameters as in figure 1. Note that, compared with figure 1 the electron intensity is revived to its maximum at $\omega t = n\pi$, n an integer, where the fluctuations of the operator f are smallest.

electron interference. The time-averaged interference fringes exist only for special values of V_0 which depends on the nature of the applied nonclassical electromagnetic field. We have found that for SN the voltage steps are the same as those for squeezed-vacuum states which are double in size in comparison with the voltage steps for coherent and squeezed states.

We have also shown that the dynamic behaviour of the electron interference exhibits CR for SN while for number state this phenomenon does not exist. CR cannot be achieved by the interaction of electrons with a classical electromagnetic field of constant amplitude and phase, nor can it be established by interaction of electrons with the particle aspects of a photon alone. We have found that CR is due to the interaction of electrons with both wave and particle aspects of photons. We recollect here that CR is a purely quantum effect which is due to the quantum nature of the field.

It is also shown that CR of electron interference is closely related to the the fluctuation of a nonclassical electromagnetic field. By studying the fluctuation of the operator $f = \mathrm{Re}\{\exp[\mathrm{i}(e\phi + eV_0t + \sigma)]\}$ we have found that the minimum fluctuations of the operator f occurs for SN when the electron intensity is revived to its maximum, and the maximum fluctuation is achieved at the collapses time where the electron intensity collapses to 1. The complete collapse leads to the complete destruction of the interference fringe. This is because the fluctuations have their origin in the destructiveness of quantum interference.

We hope the results obtained in this paper can find their applications in the future, for example, we can use the AB-type experiment to study the nonclassical electromagnetic field.

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